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Vector boson pair production in e^-e^- collisions with polarized beams

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Abstract

The W -boson pair production in e^-e^- collisions with polarized beams is investigated. The helicity amplitudes are derived for general couplings and the conditions for a good high-energy behaviour of the cross-section are given. The results are applied to the heavy vector boson production in the context of the left-right symmetric model. The Ward identities and the equivalence theorem are also discussed.

1 Introduction

While colliding electrons against positrons will be the main operation mode of the next generation linear collider (NLC), also the electron-electron option is technically viable [1]. Realizing this possibility would be well motivated from the physics point of view. The e^-e^- collisions have been so far experimentally explored only at very modest energies, where the Møller scattering dominates the total cross section. At high energies many other processes become possible providing a probe into interesting new features of the electroweak interactions [2].

The e^-e^- mode is particularly ideal for testing the possible lepton number violation, since the initial state carries, in contrast with the e^+e^- collisions, a non-vanishing lepton number (see e.g. [3, 4, 5]). It is also suitable for search of supersymmetric extensions of the electroweak models [6, 7, 8]. The possibility provided by linear colliders of having polarized beams offers effective methods for such studies. From the experimental point of view the electron-electron collision is an excellent place to study new phenomena, because the Standard Model (SM) background is low due to non-existence of dominating s-channel resonancies.

One interesting lepton number violating reaction with clear experimental characteristics is the pair production of massive vector bosons [3, 4, 9, 10]:

$$e_{L,R}^- e_{L,R}^- \rightarrow W^- W^-, \quad (1)$$

Observation of such a process would be an inevitable signal of new physics, since in the Standard Model (SM) the lepton number is conserved and the reaction (1) is absolutely forbidden (the SM reaction $e^-e^- \rightarrow W^-W^- \nu_e \nu_e$ mimicing (1) can be effectively eliminated by imposing suitable kinematical cuts [11]). In a wide class of other models beyond the SM the reaction may occur. For example, in the $SU(2)_L \times$

$SU(2)_R \times U(1)_{B-L}$ left-right symmetric theory [12] it is possible due to the lepton number violating Yukawa interactions and the existence of Majorana neutrinos [3, 4]. In that model the vector bosons may be ordinary charged weak bosons, W_1 , or the heavier right-handed weak bosons, W_2 . Depending on the assumptions made on some model parameters the cross section for the W_1 pair production may according to [9] be tens of fb's at $\sqrt{s} = 1$ TeV and some fb's at $\sqrt{s} = 0.5$ TeV. The present lower limit for the mass of W_2 is $M_{W_2} \gtrsim 600$ GeV [13] so that the reaction (1) might be kinematically allowed also for the $W_2 W_2$ final state, at least in the final phase of the NLC. Soon above the threshold the cross section of this reaction may reach the level of pb [14].

The reaction (1) can proceed at tree level through a neutral fermion exchange in t- and u-channels (see Fig. 1). As the lepton number is broken in the process, the neutral fermions exchanged have to be self-conjugated particles. Hence the reaction (1) would reveal the nature of neutrinos by making difference between the cases where neutrinos are Majorana particles and Dirac particles. Nevertheless, the amplitudes of the t and u-channels alone would violate unitarity at high energies. In order to make the total cross section consistent with unitarity also a lepton number violating doubly charged boson exchange in s-channel (see Fig. 1) has to occur and should be taken into account.

In a gauge theory the good high-energy behaviour is guaranteed by the gauge structure and both the t, u and s-channel processes are present automatically. So is the case, e.g., in the left-right symmetric model, where the doubly charged boson (Δ^{--}) is a spin-0 particle (and not a vector particle, dilepton, which appears in another class of models [5]). The Δ^{--} is a member of a right-handed Higgs triplet, which plays a central role in the model by driving the spontaneous breaking of the left-right symmetry and thereby giving a large mass to the exotic particles

predicted by the model. The triplet Higgses also give rise to the see-saw mechanism for neutrino masses [15], as a result of which there are two self-conjugate neutral fermions (neglecting the possible interfamily mixing) contributing to (1) in t- and u-channels, a light and a heavy Majorana neutrino.

In this paper we shall investigate the reaction (1) for polarized beams and with final state polarization measurements. We derive helicity amplitudes assuming a general form for the relevant couplings, and derive conditions between the s- and t- and u-channel couplings which should be satisfied in order to guarantee a good high energy behaviour of the interaction. We also study how small deviations from this condition would manifest themselves in measurable quantities. Such deviations might reflect the possibility that a gauge theory, such as the left-right theory, is just an effective theory to be replaced at short distances by a more fundamental one.

The plan of the paper is the following. In Section 2 we derive the helicity amplitudes and discuss the high energy behaviour of the cross section. In Section 3 we present numerical results. In Section 4 we consider the Ward identities and the equivalence theorem. Summary is given in Section 5.

2 Helicity amplitudes

In a gauge theory the longitudinal components of gauge bosons play a special role since they exist as a consequence of the Higgs mechanism. In the original Lagrangian they correspond to Goldstone fields, the scalar fields associated with the spontaneous breaking of the gauge symmetry. The interactions of the longitudinal gauge bosons thus reflect the properties of the Higgs scalars responsible for the symmetry breaking. It is therefore useful to investigate the reaction (1) by taking into account the polarization of the particles. In the following we give the helicity amplitudes of (1)

to serve this purpose.

Let us start by defining the relevant couplings without devoting ourselves to any particular model. We assume the interaction needed for the reaction to proceed through the neutrino exchange in t- and u-channels to have the form

$$\mathcal{L}^{cc} = \frac{g}{2\sqrt{2}} \bar{\nu} \gamma_\mu (A_1 + B_1 \gamma_5) e W^{+\mu} + h.c., \quad (2)$$

where g is a coupling constant and A_1 and B_1 are parameters. This has the same form as the weak boson - neutrino coupling in SM, except that it allows for a more general vector-axial-vector structure. For the s-channel process we have to specify the $W^- W^- \Delta^{++}$ and $e^- e^- \Delta^{++}$ interactions. The interaction between Δ^{++} and the W 's we assume to have the form

$$\mathcal{L}_{WW\Delta} = \frac{1}{\sqrt{2}} g^2 v W^- W^- \Delta^{++} + h.c. \quad (3)$$

The v can be taken as a free mass parameter making this interaction independent of the $\nu e^- W^+$ coupling.

The lepton number violating Yukawa interaction which determines the coupling between Δ and the electrons is taken to be

$$\mathcal{L}_{\nu l \Delta} = -\frac{1}{\sqrt{2}} h e^T C (A_2 + B_2 \gamma_5) \Delta^{++} e + h.c., \quad (4)$$

where h is the Yukawa coupling constant and A_2 and B_2 are free parameters.

As mentioned, using polarized beams and detecting the final state polarizations may give valuable insight into the process (1). One issue of interest in such a study would be the interactions of the longitudinal components of the vector bosons. In reality the polarization is never perfect and when calculating cross sections one needs to apply the density matrix containing the transversely and longitudinally polarized components of the beams.

We now write down the helicity amplitudes for the process (1). We will neglect the electron mass, which has the consequence that the polarized electrons coincide with the left- and right-handed chirality states denoted by $\lambda = -\frac{1}{2}$ and $\lambda = +\frac{1}{2}$, respectively. The electron momenta are assumed to be along the z -axis, $p_i = (p, 0, 0, \pm p)$ ($i = 1, 2$). The W momenta are given by $k_i^\mu = (k^0, \pm k \sin \theta, 0, \pm k \cos \theta)$ and the polarization vectors of the W 's are

$$\epsilon_{\tau=\pm 1}(k_i) = \frac{1}{\sqrt{2}}(0, \mp \tau \cos \theta, -i, \pm \tau \sin \theta), \quad (5)$$

$$\epsilon_{\tau=0}(k_i) = \pm \frac{1}{M_W}(\pm k, k^0 \sin \theta, 0, k^0 \cos \theta). \quad (6)$$

Using the notation above, the helicity amplitudes are given by

$$\begin{aligned} F_{\lambda\lambda'\tau\tau'} &= \frac{i2hg^2v}{s - M_\Delta^2} \bar{v}(p', \lambda')(A_2 + B_2\gamma_5)u(p, \lambda)\epsilon_\mu^*(k, \tau)\epsilon^{\mu*}(k', \tau') \\ &\quad - \frac{ig^2}{8} \bar{v}(p', \lambda')C(i\Gamma_{\mu_2})^T C^{-1} \frac{1}{(\not{p} - \not{k}) - m_\nu} i\Gamma_{\mu_1} u(p, \lambda) \\ &\quad \times \epsilon^{\mu_1*}(k, \tau)\epsilon^{\mu_2*}(k', \tau') + (k \leftrightarrow k'), \end{aligned} \quad (7)$$

where $\Gamma_\mu = (g/2\sqrt{2})\gamma_\mu(A_1 + B_1\gamma_5)$. The first two indices in $F_{\lambda\lambda'\tau\tau'}$ denote the electron helicities and the last two the gauge boson helicities. More explicitly, the amplitudes read ($\lambda = \pm\frac{1}{2}$ and $\tau = \pm 1$, longitudinal W 's are denoted by 0):

$$\begin{aligned} F_{\lambda-\lambda\tau\tau} &= -i \frac{g^2\lambda\sqrt{s}}{4\sqrt{s-4M_W^2}}(A_1^2 - B_1^2) \sin \theta \left\{ \frac{-t - M_W^2}{t - m_\nu^2} + \frac{-u - M_W^2}{u - m_\nu^2} \right\}, \\ F_{\lambda\lambda\tau\tau} &= -i \frac{g^2\sqrt{s}m_\nu}{8}(A_1 + 2\lambda B_1)^2 \\ &\quad \times \left\{ -\frac{1 + 2\lambda\tau \cos \theta}{t - m_\nu^2} - \frac{1 - 2\lambda\tau \cos \theta}{u - m_\nu^2} \right\} + i \frac{2\sqrt{s}g^2vh}{(s - M_\Delta^2)}(A_2 + 2\lambda B_2), \\ F_{\lambda-\lambda\tau-\tau} &= -i \frac{1}{16}g^2s\tau \sin \theta(A_1^2 - B_1^2)(1 + 2\lambda\tau \sin \theta) \left\{ \frac{1}{t - m_\nu^2} + \frac{1}{u - m_\nu^2} \right\}, \end{aligned}$$

$$\begin{aligned}
F_{\lambda\lambda\tau-\tau} &= 0, \\
F_{\lambda-\lambda\tau 0} &= -i \frac{g^2 \sqrt{s} (A_1^2 - B_1^2)}{8\sqrt{2} M_W} (1 + 2\lambda\tau \cos \theta) \\
&\quad \times \left\{ \left[-2M_W^2 \lambda\tau + \sqrt{\frac{s}{s-4M_W^2}} (t + M_W^2) \right] \frac{1}{t - m_\nu^2} \right. \\
&\quad \left. + \left[-2M_W^2 \lambda\tau - \sqrt{\frac{s}{s-4M_W^2}} (u + M_W^2) \right] \frac{1}{u - m_\nu^2} \right\}, \\
F_{\lambda-\lambda 0\tau} &= -i \frac{g^2 \sqrt{s} (A_1^2 - B_1^2)}{8\sqrt{2} M_W} (1 - 2\lambda\tau \cos \theta) \\
&\quad \times \left\{ \left[2M_W^2 \lambda\tau + \sqrt{\frac{s}{s-4M_W^2}} (t + M_W^2) \right] \frac{1}{t - m_\nu^2} \right. \\
&\quad \left. + \left[2M_W^2 \lambda\tau - \sqrt{\frac{s}{s-4M_W^2}} (u + M_W^2) \right] \frac{1}{u - m_\nu^2} \right\}, \\
F_{\lambda\lambda\tau 0} &= -i \frac{g^2 \sqrt{s} m_\nu \tau}{16\sqrt{2} M_W} (A_1 + 2\lambda B_1)^2 (-\sqrt{s-4M_W^2} - 2\lambda\tau \sqrt{s}) \sin \theta \\
&\quad \times \left(\frac{1}{t - m_\nu^2} - \frac{1}{u - m_\nu^2} \right), \\
F_{\lambda\lambda 0\tau} &= -i \frac{g^2 \sqrt{s} m_\nu \tau}{16\sqrt{2} M_W} (A_1 + 2\lambda B_1)^2 (\sqrt{s-4M_W^2} + 2\lambda\tau \sqrt{s}) \sin \theta \\
&\quad \times \left(\frac{1}{t - m_\nu^2} - \frac{1}{u - m_\nu^2} \right), \\
F_{\lambda-\lambda 00} &= -i \frac{g^2 \sin \theta (A_1^2 - B_1^2)}{16M_W^2} \sqrt{\frac{s}{s-4M_W^2}} \left\{ \frac{-st - 4M_W^4}{t - m_\nu^2} + \frac{su + 4M_W^4}{u - m_\nu^2} \right\}, \\
F_{\lambda\lambda 00} &= -i \frac{g^2 \sqrt{s} m_\nu}{8M_W^2} (A_1 + 2\lambda B_1)^2 \left\{ \frac{-t}{t - m_\nu^2} + \frac{-u}{u - m_\nu^2} \right\} \\
&\quad - i \frac{\sqrt{s} g^2 v h}{(s - M_\Delta^2) M_W^2} (A_2 + 2\lambda B_2) (s - 2M_W^2). \tag{8}
\end{aligned}$$

The t- and u-channel diagrams are proportional either to a factor $(A_1^2 - B_1^2)$ or $(A_1 + 2\lambda B_1)^2$. In the amplitudes of the latter type, there is a flip in the neutrino helicity, which is indicated by the proportionality to the neutrino mass. In order to have interaction vertices which match the helicity flip, both electrons have to be of the same chirality. Due to the annihilation into a scalar particle, the s-channel contribution is non-zero only when the W 's are similarly polarized and both electrons have the same chirality.

In case that the eW interaction is purely right-handed ($A_1 = B_1$), the t- and u-channel contributions exist only when both electrons are right-handed. For a pure left-handed interaction ($A_1 = -B_1$) the electrons should correspondingly be left-handed. The same is true for the s-channel and the couplings A_2 and B_2 . In the case of purely chiral couplings the whole process should thus disappear for suitably chosen beam polarizations. This is of course only an ideal situation, since in practice the beams will not be 100 % polarized.

In the helicity amplitudes given above we have not assumed any interrelations among the couplings or particle masses but have kept them all as free parameters. In general this would lead to a contradiction with unitarity. In order to have a cancellation of the unwanted terms proportional to s the following conditions should be fulfilled:

$$\begin{aligned} m_\nu(A_1 + B_1)^2 - 4hv(A_2 + B_2) &= 0, \\ m_\nu(A_1 - B_1)^2 - 4hv(A_2 - B_2) &= 0. \end{aligned} \tag{9}$$

This facilitates our previous statement that both the t, u and the s channel amplitudes are necessary for a good high-energy behaviour, and on top of that the neutrino mass and the couplings of the doubly charged scalar should be suitably related to each other.

3 Numerical results

In this Section we shall present our numerical results. We shall only consider the reaction $e^-e^- \rightarrow W_2^-W_2^-$, where W_2^- is a heavy gauge boson predicted by the left-right symmetric model. In the left-right symmetric model the following relation holds:

$$m_\nu = 2hv. \tag{10}$$

Here v is the vacuum expectation value of the neutral member of a "right-handed" Higgs triplet $(\Delta^{--}, \Delta^-, \Delta^0)$ which is responsible of the breaking of the $SU(2)_R \times U(1)_{B-L}$ gauge symmetry. It also determines the mass of the heavy weak boson W_2 through $M_{W_2} = gv/\sqrt{2}$, where g is the gauge coupling constant associated with the $SU(2)_R$ interactions (we shall assume it to have the same value as the SM gauge coupling, $g = 0.65$).

Our formulas are also applicable to the pair production of the ordinary weak bosons, but the cross section of this process is in general too small to have any great phenomenological importance at the energies of NLC.¹ In all plots to be presented we assume it to have the mass $M_{W_2} = 0.5$ TeV. For the mass of the doubly charged scalar Δ^{--} we shall use different values, at lowest $M_\Delta = 0.8$ TeV. Varying this mass affects the total cross section and in particular the angular distributions. In the left-right symmetric model, providing that W_1 and W_2 , as well as Δ^{--} and its possible left-handed counterpart, do not mix as we shall assume, all couplings relevant here are purely right-handed, i.e. $A_1 = B_1 = A_2 = B_2 = 1$. For this reason we shall consider the particular case where the electron beams are right-handedly polarized.

In Fig. 2 we plot the cross sections in the case of fully right-handedly polarized electron beams corresponding to various W_2 polarisation states as a function of the collision energy \sqrt{s} assuming for the neutrino mass the value $m_\nu = 1$ TeV. In Fig. 2a we assume the scalar exchanged in the s-channel to have the mass $M_\Delta = 0.8$ TeV,

¹It was argued in ref. [9] that the cross section of the pair production of the ordinary W bosons could reach some tens of fb's at $\sqrt{s} = 1$ TeV. This estimate was, however, based on a calculation where only the t- and u-channels were taken into account. If also the s-channel is included, as unitarity requires, the cancellations may reduce the cross section substantially.

in Fig. 2b $M_\Delta = 10$ TeV. The dominant contribution in both cases corresponds to the final state where both W 's are longitudinally polarized, i.e. to the helicity amplitude $F_{\frac{1}{2} \frac{1}{2} 0 0}$, and it is at the level of picobarns. The relative importance of the s-channel contribution compared with the t- and u-channel contributions depends on the masses of Δ and ν . This can be seen by comparing the plots in Figs. 2a and 2b, and by inspecting the Figs. 3a and 3b, where the cross sections are presented as a function of neutrino mass for $\sqrt{s} = 1.5$ TeV. The qualitatively different \sqrt{s} -dependence of the 00 final state cross section in Figs. 2a and 2b follows from the fact that in Fig. 2a one is above the Δ^{--} resonance and in Fig. 2b below it.

The M_Δ dependence is particularly apparent in angular distributions. In Fig. 4 we plot the differential cross sections for different WW polarization states in the collision of two right-handed electrons (we have taken $M_\nu = 1$ TeV and $M_W = 0.5$ TeV. The results corresponding to $M_\Delta = 0.8$ TeV are marked with a , and those corresponding to $M_\Delta = 10$ TeV are marked with b . The distributions for the $+1+1$, $-1-1$ and 00 final state polarizations are completely opposite in these two cases providing information about the relative importance of the various amplitudes. The other distributions do not change with M_Δ because the s-channel amplitude does not contribute to them.

We have also studied how a deviation from the purely right-handed interaction would manifest itself. Let us assume that the $e\nu W$ interaction has a small left-handed part $\epsilon\gamma_\mu(1 - \gamma_5)$. It can be studied in $e_R e_L$ collisions, which vanish for purely right-handed interactions. In Fig. 5 we have plotted the cross section for $e_R e_L \rightarrow W_{2,+1} W_{2,-1}$ as a function of the parameter ϵ . Taking ten events as discovery limit, one finds that NLC with its anticipated luminosities of $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ would be sensitive to the deviations corresponding to $\epsilon \gtrsim 0.03$.

4 Ward identities and equivalence theorem

According to the well known equivalence theorem [16, 17] the S-matrix element involving longitudinally polarized vector bosons is at high energies up to a small correction the same as the S-matrix element obtained by replacing the vector bosons by the corresponding unphysical Goldstone bosons. In this Section we will consider this theorem by investigating the pair production of longitudinally polarized W_2 bosons:

$$e^- e^- \rightarrow W_{2,\text{long}} W_{2,\text{long}}. \quad (11)$$

The Goldstone boson corresponding to $W_{2,\text{long}}$ is the predominantly singly charged member of the Higgs triplet, Δ^- (assuming no mixing of W_1 and W_2). The s-channel amplitude for (11) is hence directly controlled by the triplet Higgs self-interactions giving a valuable insight to the symmetry breaking sector of the model.

The Ward identities for the process (11) take the form [16]

$$\frac{p_1^{\mu_1} p_2^{\mu_2}}{M_{W_2}^2} S_{\mu_1 \mu_2}(p_1, p_2) - S_{44}(p_1, p_2) + i \frac{p_1^{\mu_1}}{M_{W_2}} S_{\mu_1 4}(p_1, p_2) + i \frac{p_2^{\mu_2}}{M_{W_2}} S_{4 \mu_2}(p_1, p_2) = 0, \quad (12)$$

$$i \epsilon_{(L)}^{\mu_1}(p_1) \frac{p_2^{\mu_2}}{M_W} S_{\mu_1 \mu_2}(p_1, p_2) - \epsilon_{(L)}^{\mu_1}(p_1) S_{\mu_1 4}(p_1, p_2) = 0. \quad (13)$$

Here $p_1^{\mu_1}$ and $p_2^{\mu_2}$ are the four-momentas of the right handed vector or Goldstone bosons. The notation is such that $\epsilon_1^{\mu_1} \epsilon_2^{\mu_2} S_{\mu_1 \mu_2}(p_1, p_2)$ is the S-matrix element of the process $e^- e^- \rightarrow W^- W^-$, $\epsilon_1^{\mu_1} S_{\mu_1 4}(p_1, p_2)$ is the S-matrix element of the process $e^- e^- \rightarrow W^- \Delta^-$ and $S_{44}(p_1, p_2)$ is the S-matrix element of the process $e^- e^- \rightarrow \Delta^- \Delta^-$.

Expressing the longitudinal polarization vector of a vector boson with momentum p as

$$\epsilon_{(L)}^\mu(p) = \frac{p^\mu}{M_W} + v^\mu(p), \quad (14)$$

where $v^\mu(p)$ is a four-vector with the components of the order M_W/E , one can, with help of the equation (13), cast the equation (12) to the form

$$S_{44}(p_1, p_2) = -\epsilon_{(L)}^{\mu_1}(p_1)\epsilon_{(L)}^{\mu_2}(p_2)S_{\mu_1\mu_2}(p_1, p_2) + \mathcal{O}(\frac{M_{W_2}}{E}). \quad (15)$$

This is the equivalence theorem for our case.

The processes involving Goldstone bosons relevant for us are

$$e^-e^- \rightarrow \Delta^- \Delta^- \quad (16)$$

and

$$e^-e^- \rightarrow W^- \Delta^-. \quad (17)$$

The Feynman diagrams of these reactions are given in Figs. 6 and 7. The relevant Feynman rules follow from the kinetic Lagrangian,

$$\mathcal{L} = (D_\mu \Delta_R)^\dagger (D^\mu \Delta_R), \quad (18)$$

from the scalar potential

$$V = -\mu^2 \text{Tr}(\Delta_R \Delta_R^\dagger) + \rho_1 (\text{Tr}(\Delta_R \Delta_R^\dagger))^2 + \rho_2 \text{Tr}(\Delta_R \Delta_R) \text{Tr}(\Delta_R^\dagger \Delta_R^\dagger), \quad (19)$$

and from the Yukawa coupling

$$\mathcal{L}_Y = -\frac{1}{\sqrt{2}} h \psi_l^T C (A_2 + B_2 \gamma_5) \Delta \psi_l + h.c., \quad (20)$$

where $\psi_l = (\nu, l^-)$.

A straightforward calculation gives for the helicity amplitude of the process (16) the expression:

$$F_{\lambda\lambda}^{\Delta\Delta} = -i4h^2 m_\nu \sqrt{s} \left\{ \frac{1}{(t - m_\nu^2)} + \frac{1}{(u - m_\nu^2)} \right\} + i \frac{32\rho_2 \sqrt{s} h v}{(s - M_\Delta^2)}. \quad (21)$$

According to the equivalence theorem (15) this should approach in the limit $s \rightarrow \infty$ the helicity amplitude $F_{\lambda\lambda 00}$ with the reversed sign. Indeed, replacing h, v and ρ_2

by their left-right model expressions from the formulas $M_W^2 = g^2 v^2 / 2$, $M_\Delta^2 = 8\rho_2 v^2$ and $m_\nu = 2hv$ we see that this is the case. However, it is worth of noticing that the equivalence theorem is applicable for approximations only if not just the mass of final state vector bosons but also that of the virtual doubly charged scalar is small compared with the collision energy.

Let us now turn to the Ward identity (12). In order to make use of this identity we have to calculate quantities $(p_1^{\mu_1} p_2^{\mu_2} / M_W^2) S_{\mu_1 \mu_2}(p_1, p_2)$ and $(p_1^{\mu_1} / M_W) S_{\mu_1 4}(p_1, p_2)$. The first of them turns out to be equal to the helicity amplitude $F_{\lambda\lambda 00}$ given in (8). The second quantity denoted as $F_{\lambda\lambda}^{\Delta W}$ is given by

$$F_{\lambda\lambda}^{\Delta W} = -\frac{\sqrt{2}gh\sqrt{s}}{M_W} \left\{ \frac{t}{(t - m_\nu^2)} + \frac{u}{(u - m_\nu^2)} - 2\frac{(s - M_W^2)}{(s - M_\Delta^2)} \right\}. \quad (22)$$

Obviously $F_{\lambda\lambda}^{W\Delta} = F_{\lambda\lambda}^{\Delta W}$.

The identity (12) can be used, at least in principle, for extracting information about the triplet Higgs potential (19). Discovery of the process $e^- e^- \rightarrow W_2^- W_2^-$ would enable one to measure the mass of the gauge boson and thus also the value of the vacuum expectation value of Δ^0 determined by μ^2 and ρ_1 . By studying other lepton number violating processes like $e^- e^- \rightarrow \mu^- \mu^-$ [18] and also $e^- e^+ \rightarrow W^- W^+$ [4] one could determine the gauge and Yukawa coupling constants g and h , as well as the masses M_Δ and m_ν . Using these experimental data one can cross check the model.

In order to get a feeling how sensitive such a test could be, let us violate the gauge model relation (10) by writing it as

$$m_\nu = 2(1 + \delta)hv, \quad (23)$$

where δ is a parameter. Then the Ward identity (12) takes the form

$$\frac{g^2}{2(1+\delta)} \left\{ \frac{(1-\delta)^2 t - m_\nu^2}{t - m_\nu^2} + \frac{(1-\delta)^2 u - m_\nu^2}{u - m_\nu^2} \right\} - \frac{g^2 s}{s - M_\Delta^2} + \frac{16\rho_2 M_{W_2}^2}{s - M_\Delta^2} = 0. \quad (24)$$

Let us emphasize that in this expression all the parameters except ρ_2 have their experimentally measured values. Therefore, together with the kinematical constraint $s + t + u = 2M_{W_2}^2$, the formula (24) determines the parameter ρ_2 .

In Fig. 8 we present the dependence of ρ_2 on M_Δ for the three different values of the parameter δ : a) $\delta = -0.2$, b) $\delta = 0$ and c) $\delta = 0.2$ at the fixed $\sqrt{s} = 1.5$ TeV, $M_W = 0.5$ TeV and $m_\nu = 1$ TeV. The case $\delta = 0$ corresponds to the left-right model and the Ward identity (24) just reproduces the gauge theory relation between masses, $\rho_2 = g^2 M_\Delta^2 / (16M_{W_2}^2)$.

5 Summary

The next generation linear colliders will provide us a possibility to study high-energy electron-electron collisions in any combinations of polarization. This will be very useful option in studying new physics because of a low background from the Standard Model phenomena. In this paper we have studied the reaction $e^-e^- \rightarrow W^-W^-$ where W is a charged vector boson. Starting with arbitrary particle masses and a quite general form for the couplings involved, we have derived the helicity amplitudes of the reaction. We have also given the conditions for a good high-energy behaviour of the cross section. We have explored how one could get information about the underlying theory by using polarized beams and measurement of the vector boson helicities.

Our numerical results concern the pair production of heavy vector bosons, W_2 ,

in the framework of the left-right symmetric model. The cross section is dominated by the longitudinally polarized W_2 's. Its value depends on the masses of W_2 , the heavy Majorana neutrino and the doubly charged Higgs boson, but for a feasible choice it is on the level of a few picobarns. As longitudinal components correspond to the Goldstone bosons, the measurement of this process would give an insight to the breaking of the left-right symmetry. We also discuss the Ward identities and the equivalence theorem, which also are useful in studying the symmetry breaking sector of the left-right model.

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Figure captions

Figure 1: Feynman diagrams for the process $e^-e^- \rightarrow W^-W^-$.

Figure 2: The total cross section of various $W_2^-W_2^-$ polarization states for fully right-handedly polarized beams as a function of collision energy \sqrt{s} . The mass of W_2 is taken to be $M_W = 0.5$ TeV, the mass of heavy neutrino $m_\nu = 1$ TeV and the mass of doubly charged Higgs boson a) $M_\Delta = 0.8$ TeV and b) $M_\Delta = 10$ TeV.

Figure 3: The total cross section of various $W_2^-W_2^-$ polarization states for fully right-handedly polarized beams as a function of neutrino mass m_ν . The collision energy is $\sqrt{s} = 1.5$ TeV, the mass of W_2 is taken to be $M_W = 0.5$ TeV and the mass of doubly charged Higgs boson a) $M_\Delta = 0.8$ TeV and b) $M_\Delta = 10$ TeV.

Figure 4: The angular distribution of differential cross section of various $W_2^-W_2^-$ polarization states for fully right-handedly polarized beams. The mass of W_2 is taken to be $M_W = 0.5$ TeV, the mass of heavy neutrino $m_\nu = 1$ TeV and the mass of doubly charged Higgs boson a) $M_\Delta = 0.8$ TeV and b) $M_\Delta = 10$ TeV. The off-diagonal distributions do not depend on M_Δ .

Figure 5: The total cross section of the process $e_R^-e_L^- \rightarrow W_{+1}^-W_{-1}^-$ as a function of parameter ϵ , the fraction of the V-A coupling.

Figure 6: Feynman diagrams for the process $e^-e^- \rightarrow \Delta^-W^-$.

Figure 7: Feynman diagrams for the process $e^-e^- \rightarrow \Delta^-\Delta^-$.

Figure 8: The dependence of the parameter ρ_2 on the mass of doubly charged Higgs boson for the three values of parameter δ : a) $\delta = -0.2$, b) $\delta = 0$ and c) $\delta = 0.2$. Here δ measures the deviation of the heavy neutrino mass from its left-right model value.

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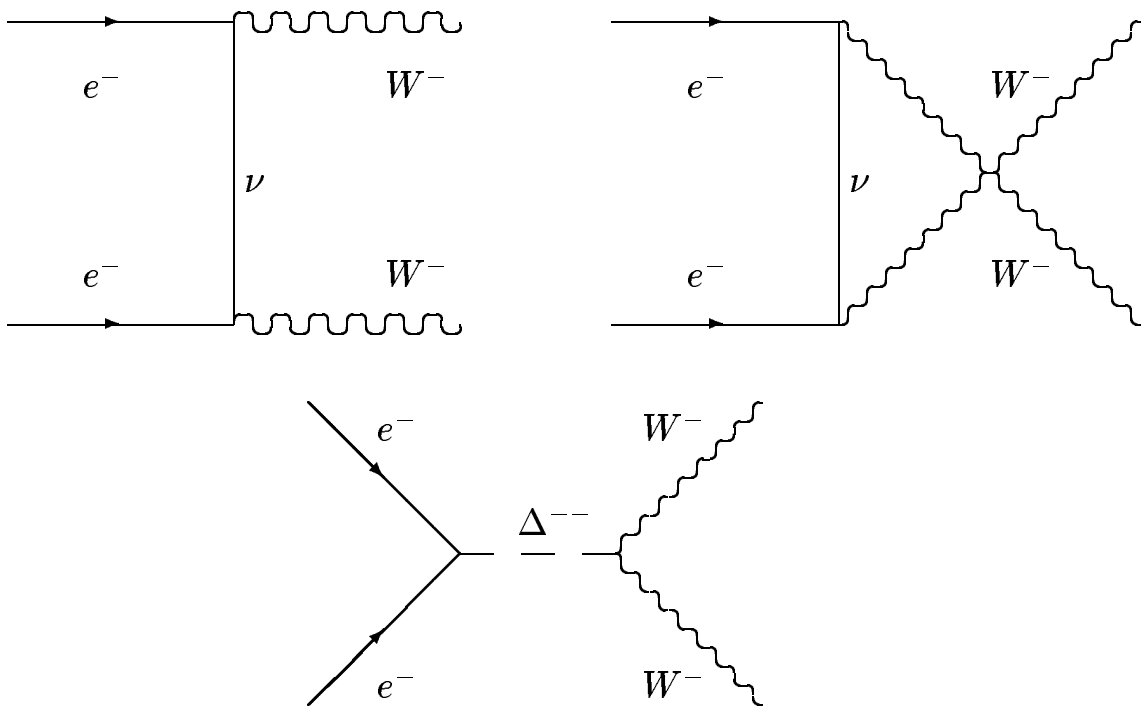


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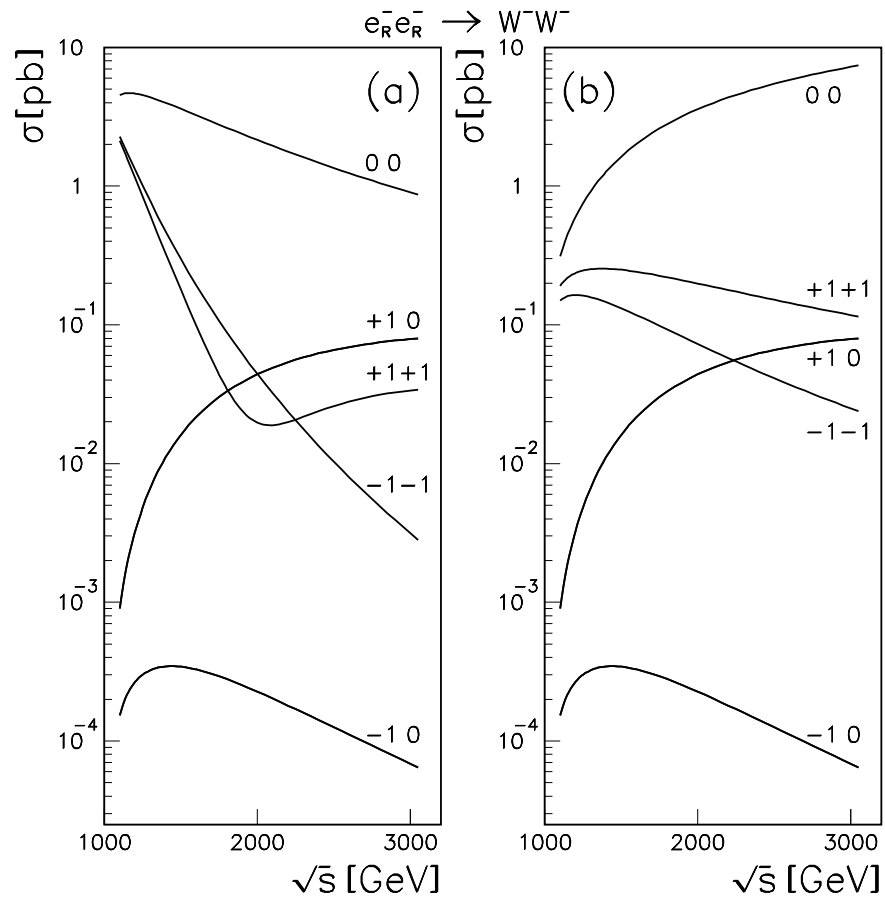


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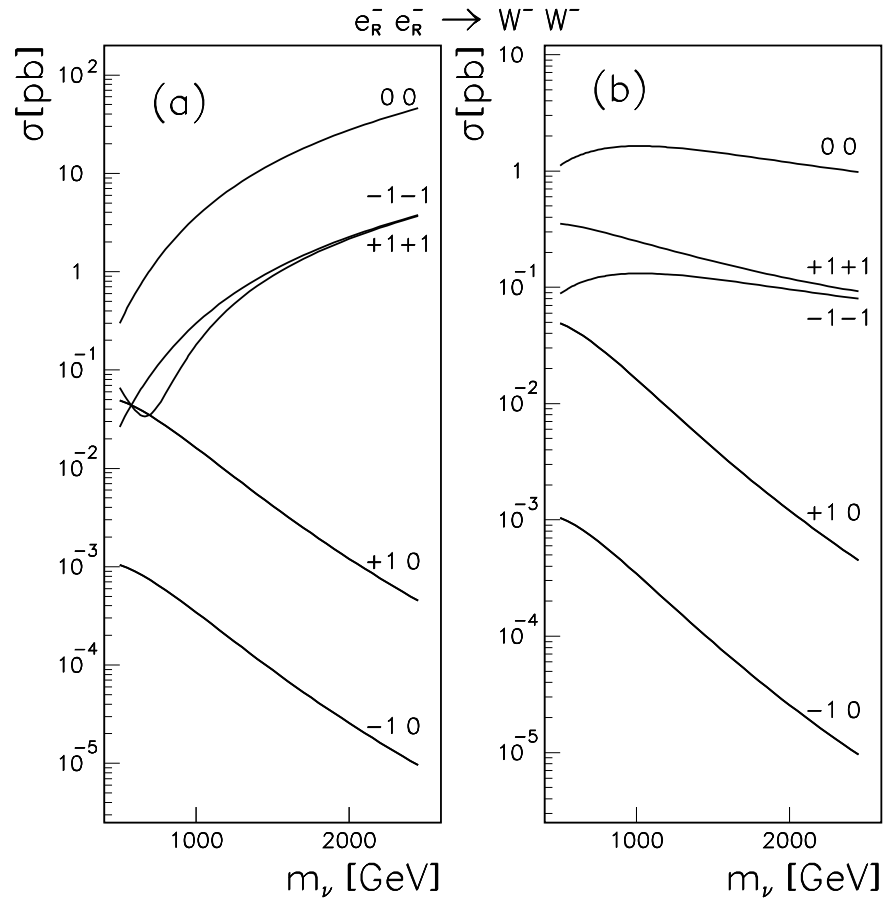


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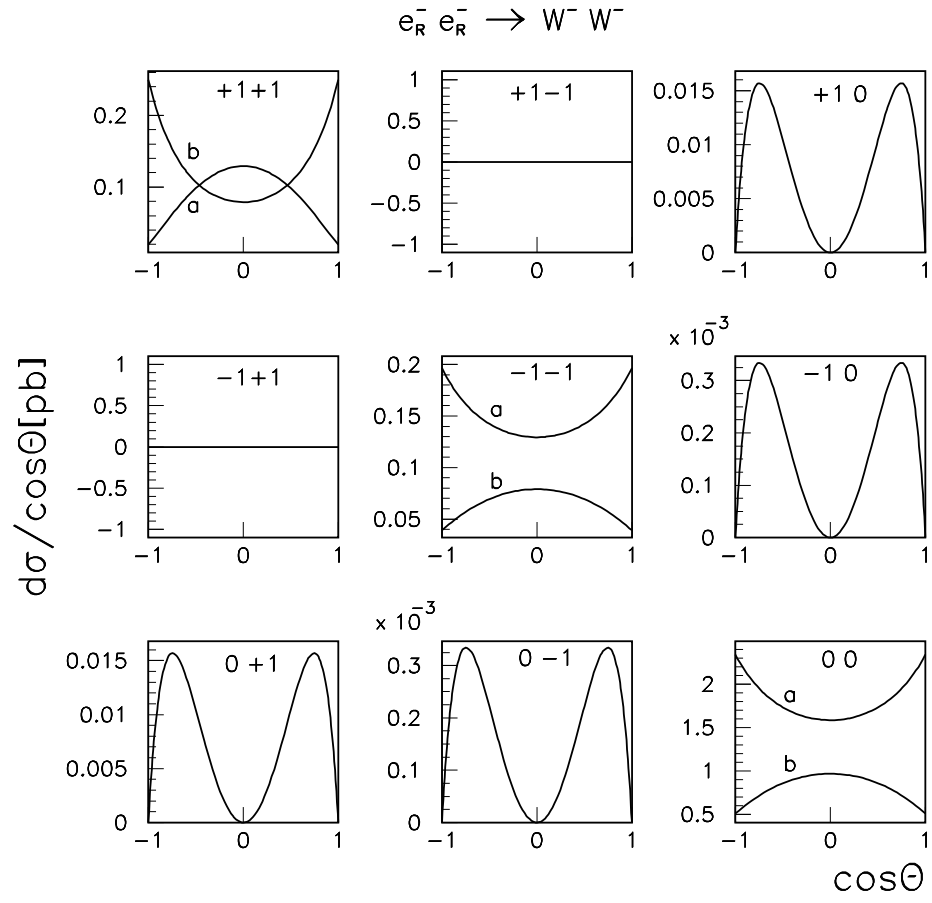


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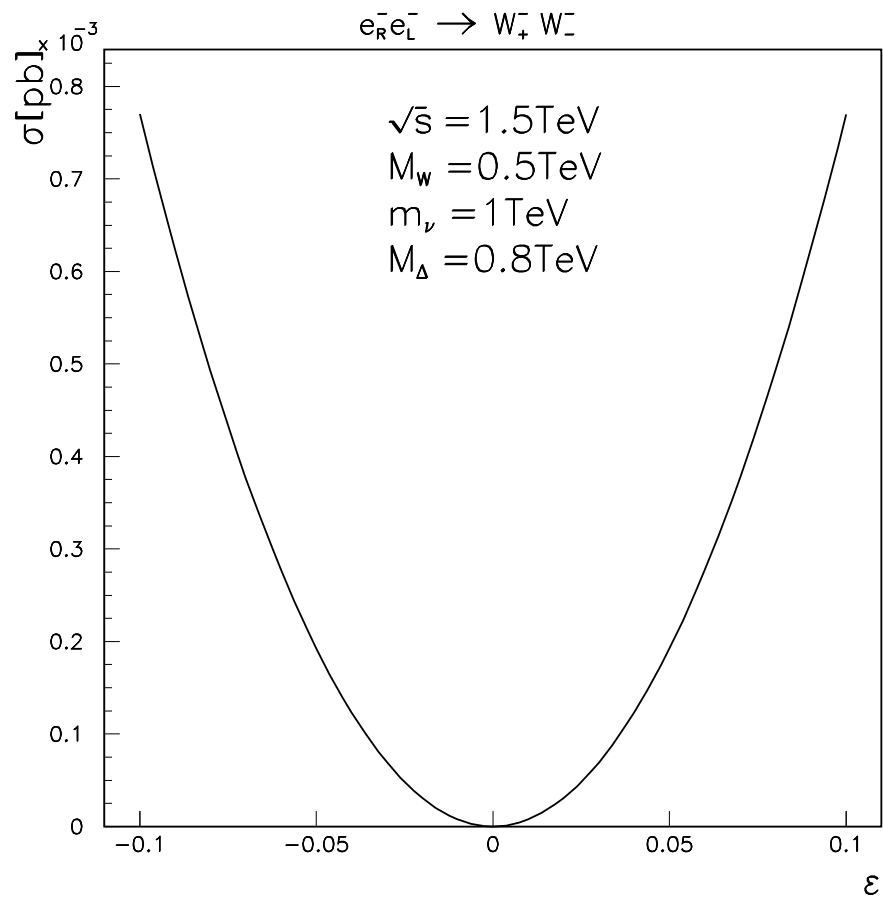


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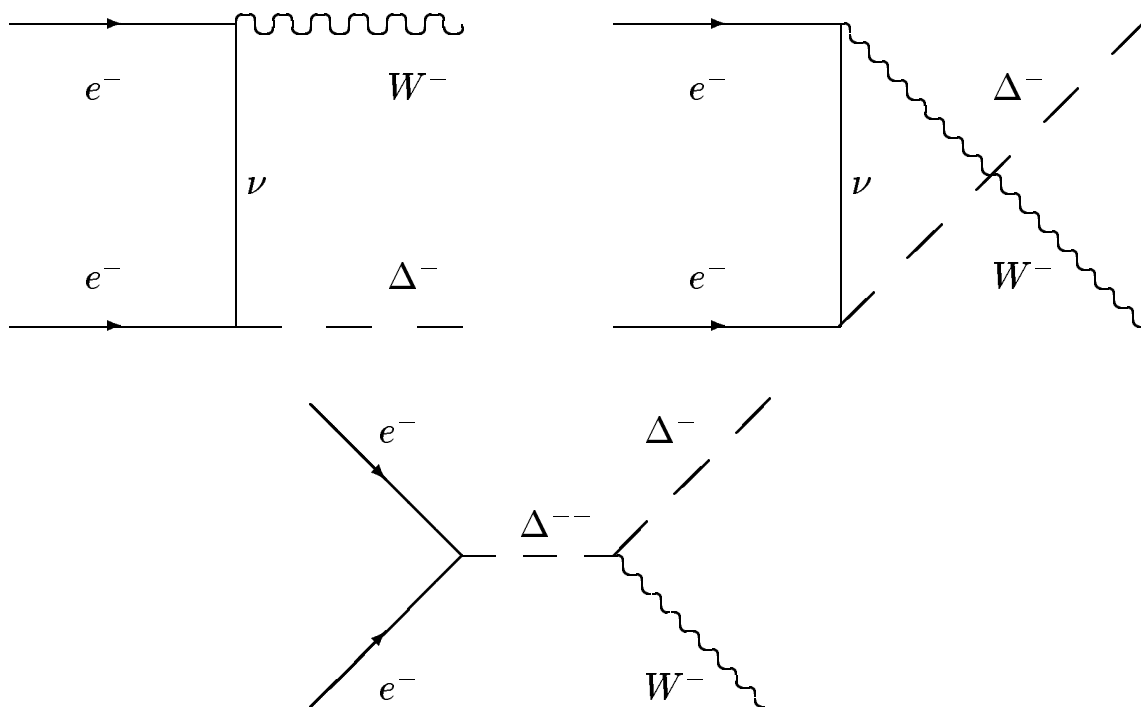


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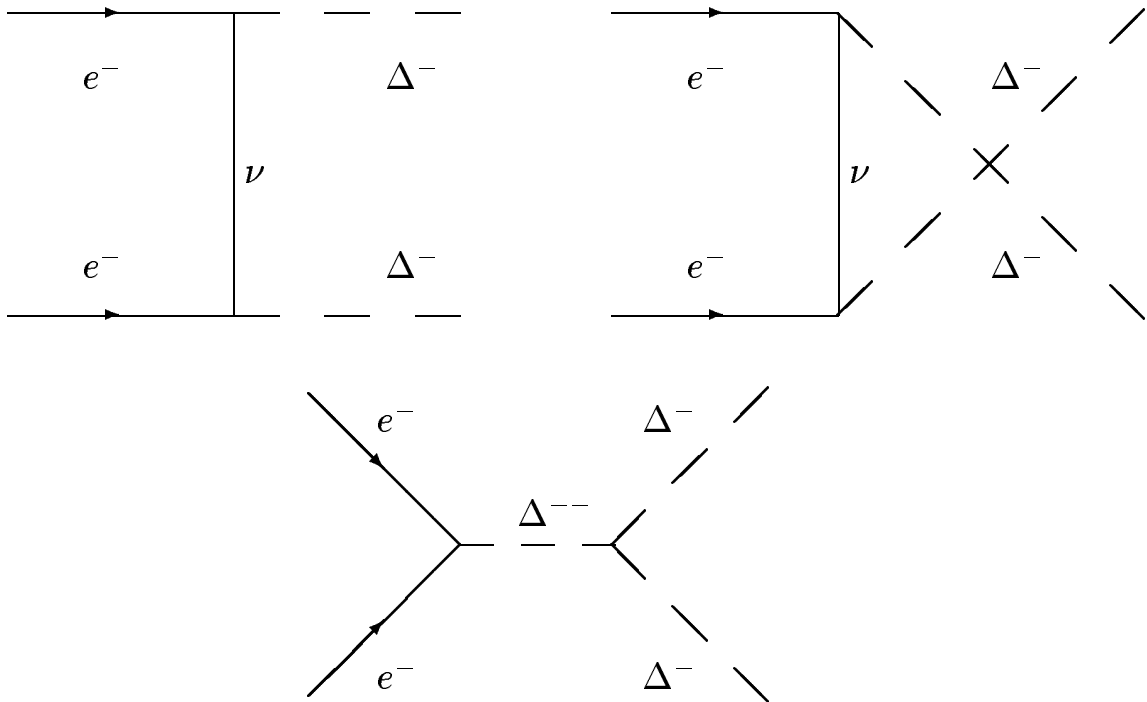


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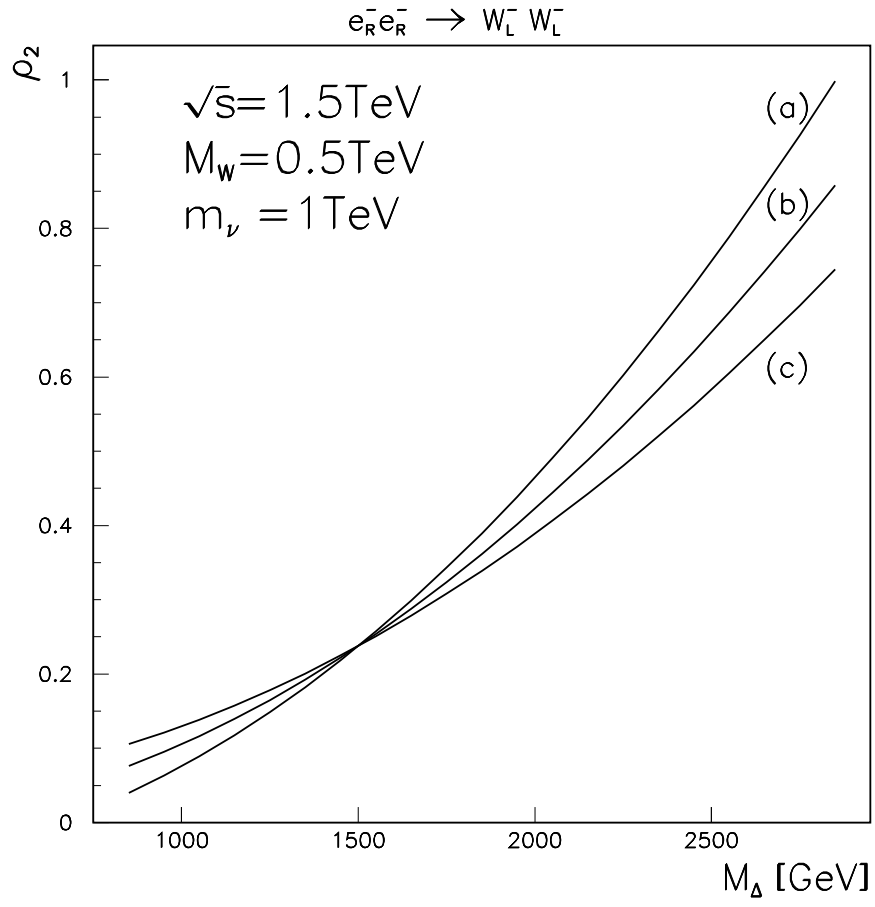


Figure 8: